Binding Energy of a Λ Particle in Nuclear Matter

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The binding energy of a Λ particle in nuclear matter, $B_\Lambda(\infty)$, is calculated using a Λ-nucleon two-body potential with a hard core, which reproduces the binding energies of light hypernuclei and the Λ-nucleon scattering at intermediate energies. The simplified version of the Brueckner theory used in previous calculations is applied. The effective mass of the Λ particle, M_Λ^*, is estimated to be about 0.9 M_Λ. The rearrangement energy is included in the calculation. The result obtained, $B_\Lambda(\infty)\approx 31$ MeV, is in good agreement with the measured value.

I. INTRODUCTION

SINCE the first observation of a hyperfragment by Danysz and Pniewski,[1] there have been many attempts to determine the parameters of the Λ-nucleon interaction from the measured binding energies of hyperfragments. In most of these attempts the Λ-nucleon interaction represented by an effective two-body central potential has been used in variational calculations of the binding energies of light hypernuclei. These energies, however, are determined primarily by the S-wave interaction. Hence, these calculations led to specification of the Λ-nucleon potential in S state only (see, for example, the report by Dalitz[2]).

Information about the Λ-nucleon interaction in higher angular momentum states can be obtained by analyzing the binding energies of heavy hypernuclei which do depend on the interaction in these states.

Instead of calculating the binding energy of a Λ particle in a heavy but finite nucleus, it is easier to calculate the binding energy in an infinite nuclear medium, i.e., in nuclear matter. To determine empirically the binding energy of a Λ particle in nuclear matter, one can procede in one of the following two ways:

(i) One represents the ${}_\Lambda Z^A$ hypernucleus (A = total mass number which includes the one Λ particle) by a single-particle potential $u(r)$ in which the Λ particle moves. This potential contains two parameters: the radius $R=r_0(A-1)^{1/3}$ (r_0 is known, e.g., from electron scattering experiments) and the depth U ($U<0$). For a given analytical form of u, one solves the Schrödinger equation for the Λ particle moving in the potential $u(r)$ and determines the lowest energy eigenvalue $E_0=E_0(A,U)$. By comparing $E_0(A,U)$ with the measured binding energies, $B_\Lambda(A)$, for different hypernuclei ${}_\Lambda Z^A$, one can determine the value of U which gives the best over-all agreement between $E_0(A,U)$ and $-B_\Lambda(A)$. Obviously, we have

$$U=\lim_{A\to\infty}E_0(A,U)=-\lim_{A\to\infty}B_\Lambda(A)\,. \qquad (1)$$

Hence, $-U$ is equal to $B_\Lambda(\infty)$, the binding energy of a Λ particle in nuclear matter.[3,4]

(ii) One can simply extrapolate the measured values of $B_\Lambda(A)$ for $A\to\infty$.[5]

Both of the methods give the result[6]

$$B_\Lambda(\infty)\cong 30 \text{ MeV}. \qquad (2)$$

Several calculations of $B_\Lambda(\infty)$ have been published.[3,7–10] We shall concentrate our attention on I and II which seem to be the most reliable calculations.[11] The Λ-nucleon potential considered in I and II is assumed to have a hard core. Hence, one applies the Brueckner theory in calculating U. In I and II, the simplest version of the Brueckner theory has been applied, namely that of Gomes, Walecka, and Weisskopf[12] This seems to be well justified because up to now we know very little about the details of Λ-nucleon interaction and it would be premature to get involved in any more extensive computations. Furthermore, in a calculation of the binding energy of a pure nuclear matter, one needs a high degree of accuracy because the potential part of the energy is nearly cancelled by the kinetic part. However, in the case of the single-particle energy of a Λ particle in its lowest state in nuclear matter, the kinetic energy of the Λ particle is zero. Therefore, a less accurate calculation of the potential part seems to be justified.

The results of I and II are as follows: If one fixes the parameters of the Λ-nucleon potential in the S state to get the proper binding of the light hypernuclei, and assumes that the same potential acts in higher angular

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[1] M. Danysz and J. Pniewski, Phil. Mag. 44, 348 (1953).

[2] R. H. Dalitz, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Heywood and Company, Ltd., London, 1961), p. 103.

[3] J. D. Walecka, Nuovo Cimento 16, 342 (1960).

[4] J. W. Olley, Australian J. Phys. 14, 313 (1961).

[5] D. H. Davis, R. Levi Setti, M. Raymund, T. Skjeggestad, G. Tomasini *et al.*, Phys. Rev. Letters 9, 464 (1962).

[6] A full discussion of the empirical estimates of $B_\Lambda(\infty)$ is given in Refs. 9 and 10.

[7] M. Taherzadeh, S. A. Moszkowski, and P. C. Sood, Nuovo Cimento 23, 168 (1962).

[8] A. R. Bodmer and S. Sampanthar, Nucl. Phys. 31, 251 (1962).

[9] B. W. Downs and W. E. Ware, Phys. Rev. 133, B134 (1964); hereafter referred to as I.

[10] B. Ram and B. W. Downs, Phys. Rev. 133, B420 (1964); hereafter referred to as II.

[11] We shall not discuss here the perturbation treatment of Ref. 8 which cannot be applied in the case of realistic Λ-nucleon interaction containing a hard core.

[12] L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Ann. Phys. (N. Y.) 3, 241 (1958).

momentum states one gets $B_\Lambda(\infty)\cong 40$ MeV. This result is too big compared to the experimental value (around 30 MeV). As a possible explanation of this discrepancy, a suppression of the interaction in higher l states has been suggested in an early paper by Walecka.[3]

Obviously, the best direct source of information about the Λ-nucleon interaction would be Λ-nucleon scattering. The few available experimental data on Λ-nucleon scattering cross section have been analyzed in II with the help of a Λ-nucleon potential with a flexible suppression of the interaction in higher l states. The results show that one gets clearly the best agreement with the experimental scattering data in the case of no suppression of the Λ-nucleon interaction in higher l states. The problem then arises of how to explain the discrepancy between the calculated and measured value of $B_\Lambda(\infty)$. As a possible explanation, the role of many-body forces is mentioned in II.

However, even in the simplified version of the Brueckner theory applied in I and II, there are two effects which should be considered: (1) the fact that the effective mass M_Λ^* of the Λ particle is smaller than its real mass M_Λ; (2) the rearrangement effects.

In all the previous calculations the assumption $M_\Lambda^*=M_\Lambda$ has been made. Two reasons for this assumption have been given[3]: First, there are no exchange integrals arising from the Pauli principle and, second, the Λ-nucleon interaction assumed does not contain space exchange force. However, one of the causes for M_Λ^* being smaller than M_Λ is the repulsive core in the Λ-nucleon interaction. A simple estimate of the reduction of M_Λ^* caused by the repulsive core leads to a reduced value of $B_\Lambda(\infty)$.

Most important, however, is the rearrangement effect. In all the previous calculations, the single-particle model potential V has been identified with $-B_\Lambda(\infty)$. The single-particle model potential V is introduced in the Brueckner theory for the sole purpose of calculating the total energy of the system (to cancel higher order graphs) and is not equal to the potential part of the separation energy.[13] Let us consider the case of a particle at the bottom of the Fermi sea which then does not have any kinetic energy. To separate this particle from the system we first have to perform the work equal to $-V$. However, the system left with the hole at the bottom of the Fermi sea has now the possibility to rearrange itself to an energetically more favorable state, and while doing it releases the rearrangement energy V_R. Hence, the separation (or binding) energy $B=-(V+V_R)=-U<-V$.

The problem of the rearrangement energy has been recognized for a long time in the theory of nuclear matter.[13] In particular, Brueckner and his collaborators[14,15] have calculated V_R splitting it into two parts. One is the exclusion contribution, V_R(exclusion), connected with the change in the operation of the Pauli principle when a hole is created by removing a nucleon. This part is absent in the case of a Λ particle in nuclear matter as Λ is not an identical particle with nucleons. The other part, the effective-mass contribution $V_R(M^*)$, is connected with the change in the single-particle model energies (or equivalently, in the effective mass M^*) caused by the removal of one particle from the system. In the case of a nucleon in nuclear matter removed from the bottom of the Fermi sea, the calculation of Ref. 15 gave $V_R(M^*)\cong 10$ MeV. An effect of the same order of magnitude should be expected also in the case of a Λ particle.

In the present paper we calculate the binding energy of a Λ particle in nuclear matter. We apply the approximate procedure of I and II but include the change in the effective mass of the Λ particle M_Λ^* and the rearrangement effect. We find that these two effects reduce the calculated value of $B_\Lambda(\infty)$ to a value which lies within the range of the experimental estimates.

It should be noticed that from the point of view of the many-body theory, the case of a Λ particle in nuclear matter presents an interesting case in which only one part of the rearrangement energy, namely $V_R(M^*)$, appears. Also, the simple method of calculating $V_R(M^*)$ presented in this paper is not quite standard. It has been first applied in Ref. 16 in connection with the theory of a finite nucleus.

All the calculations of the present paper assume the following value for the density of nuclear matter:

$$\rho=A/\Omega_V=0.172 \text{ nucleons/F}^3. \tag{3}$$

The corresponding values of the Fermi momentum $\hbar k_F$ and the parameter r_0, connected with ρ by the equation

$$\rho=(\tfrac{4}{3}\pi r_0^3)^{-1}=2k_F^3/3\pi^2 \tag{4}$$

are

$$k_F=1.366 \text{ F}^{-1}, \quad r_0=1.113 \text{ F}. \tag{5}$$

In all our calculations, we assume the Λ-nucleon potential $v_{\Lambda N}$ to be spin-independent and to be the same in all angular momentum states.

$$v_{\Lambda N}(r)=v_{\Lambda C}(r)+v_{\Lambda\Lambda}(r), \tag{6}$$

where

$$v_{\Lambda C}(r)=\begin{cases}\infty & \text{for } r<c,\\ 0 & \text{for } r>c,\end{cases} \tag{6a}$$

$$v_{\Lambda\Lambda}(r)=\begin{cases}0 & \text{for } r<c,\\ -U_0 & \text{for } c<r<c+b,\\ -W\exp[-2(r-c)/R] & \text{for } r>c+b,\end{cases} \tag{6b}$$

[13] K. A. Brueckner, Phys. Rev. **110**, 597 (1958); N. M. Hugenholtz and L. Van Hove, Physica **24**, 363 (1958).

[14] K. A. Brueckner and D. T. Goldman, Phys. Rev. **117**, 207 (1960).

[15] K. A. Brueckner, J. L. Gammel, and J. T. Kubis, Phys. Rev. **118**, 1438 (1960).

[16] H. S. Köhler, Nucl. Phys. **38**, 661 (1962).

with

$$c=0.4\text{ F},\quad R=0.847\text{ F},\quad W=150\text{ MeV}. \tag{7}$$

The following values of b and U_0 are considered:

$$b=\left\{\begin{matrix}0.7\\1.1\\1.5\end{matrix}\right\}\text{F},\quad U_0=\left\{\begin{matrix}51.3\\27.6\\16.1\end{matrix}\right\}\text{MeV}. \tag{8a, 8b, 8c}$$

The potential given by Eqs. (6)–(8) has been considered in II. It gives a reasonable agreement with the few measured Λ-nucleon scattering cross sections and its S-wave part reproduces the binding energies of the light hyperfragments. Actually, the values of U_0, Eq. (4), are the spin-averaged values. Instead of considering different values of U_0 in the singlet and triplet state of the Λ-nucleon system, it is simpler to deal with a spin-independent potential with a properly averaged value of U_0. The result is the same since in our calculations we shall restrict ourselves to terms linear in the attractive part of $v_{\Lambda N}$.

As in I and II, we shall use the following simplified form of the nucleon-nucleon interaction used in Ref. 12:

$$v_{NN}=v_N(r)(1+P_r)/2, \tag{9}$$

where P_r is the space exchange operator, and

$$v_N(r)=v_{NC}(r)+v_{NA}(r), \tag{10a}$$

and where

$$v_{NC}(r)=\begin{cases}\infty & \text{for } r<c_N,\\ 0 & \text{for } r>c_N\end{cases} \tag{10b}$$

$$v_{NA}(r)=\begin{cases}0 & \text{for } r<c_N,\\ -V_N & \text{for } c_N<r<b_N+c_N,\\ 0 & \text{for } b_N+c_N<r,\end{cases} \tag{10c}$$

with

$$V_N=\hbar^2\pi^2/4M_Nb_N^2,\qquad b_N=1.9\text{ F},\quad c_N=c=0.4\text{ F}. \tag{11}$$

The nucleon-nucleon interaction, Eqs. (9)–(11), enters into our calculations in two ways. Firstly, the effective nucleon mass M_N^* is determined by v_{NN}. Its value at the density given by Eq. (3) has been calculated in I to be

$$M_N^*=0.735M_N. \tag{12}$$

Secondly, v_{NN} enters explicitly into the expression of the rearrangement energy.

II. GENERAL EXPRESSION FOR THE BINDING ENERGY OF A Λ PARTICLE IN NUCLEAR MATTER

We shall consider a nuclear matter with equal number of protons and neutrons ($Z=N=A/2$). We also assume that the number of protons (neutrons) with spin up is the same as the number of protons (neutrons) with spin down.

By $E(A)$ we denote the ground-state energy of nuclear matter, and by $E(A+1_\Lambda)$, the ground-state energy of the system: nuclear matter +Λ particle.

The binding energy of the Λ particle $B_\Lambda(\infty)$ is given by

$$-B_\Lambda(\infty)=U=E(A+1_\Lambda)-E(A). \tag{13}$$

Both the energies $E(A+1_\Lambda)$ and $E(A)$ consist of a potential and a kinetic part. In the ground state of the nuclear matter +Λ-particle system, the Λ particle occupies the state with zero momentum. Hence, we have

$$U=E_{\rm POT}(A+1_\Lambda)-E_{\rm POT}(A). \tag{14}$$

According to the Brueckner theory, we have

$$E_{\rm POT}(A)=\sum_{<k_F}\sum_{m_1}\sum_{<k_F}\sum_{m_2}\sum_S\sum_T(2S+1)(2T+1)\times(\mathbf{m}_1\mathbf{m}_2|K(A)|\mathbf{m}_1\mathbf{m}_2), \tag{15}$$

where $K(A)$ is the reaction matrix for nucleon-nucleon interaction in nuclear matter, S is the total spin, and T the total isospin of the two interacting nucleons. The momenta of the two nucleons are $\hbar\mathbf{m}_1$ and $\hbar\mathbf{m}_2$. The usual factor $\frac{1}{2}$ is cancelled in Eq. (15) by the factor 2 introduced by the exchange term.

For our simplified nucleon-nucleon potential, Eq. (9), $K(A)$ is spin-independent. Furthermore, since the potential acts only in even l states, the total isotopic spin T is determined by S ($T=0$ for $S=1$, and $T=1$ for $S=0$). Hence, we have

$$E_{\rm POT}(A)=6\sum_{<k_F}\sum_{m_1}\sum_{<k_F}\sum_{m_2}(\mathbf{m}_1\mathbf{m}_2|K(A)|\mathbf{m}_1\mathbf{m}_2). \tag{16}$$

Similarly, we have

$$E_{\rm POT}(A+1_\Lambda)=6\sum_{<k_F}\sum_{m_1}\sum_{<k_F}\sum_{m_2}(\mathbf{m}_1\mathbf{m}_2|K(A+1_\Lambda)|\mathbf{m}_1\mathbf{m}_2)+4\sum_{<k_F}\sum_{m_1}(\mathbf{m}_1\mathbf{m}_\Lambda|\mathcal{K}|\mathbf{m}_1\mathbf{m}_\Lambda), \tag{17}$$

where $K(A+1_\Lambda)$ is the reaction matrix for nucleon-nucleon interaction in the nuclear matter +Λ-particle system, and where $\mathcal{K}$ is the reaction matrix for Λ-nucleon interaction which is assumed to have the simple form (6). The wave vector of the Λ particle is denoted by $\mathbf{m}_\Lambda$ (in our case $\mathbf{m}_\Lambda=0$).

Inserting Eqs. (16) and (17) into Eq. (14) we get

$$U=V+V_R, \tag{18}$$

where the single-particle model potential V is given by

$$V=4\sum_{<k_F}\sum_{m_1}(\mathbf{m}_1\mathbf{m}_\Lambda|\mathcal{K}|\mathbf{m}_1\mathbf{m}_\Lambda), \tag{19}$$

and the rearrangement potential V_R is

$$V_R=6\sum_{<k_F}\sum_{m_1}\sum_{<k_F}\sum_{m_2}(\mathbf{m}_1\mathbf{m}_2|K(A+1_\Lambda)-K(A)|\mathbf{m}_1\mathbf{m}_2). \tag{20}$$

A. Expression for V

Let us now describe briefly the approximate calculation of V presented in I. First, one goes over to the relative coordinates since the $\mathcal{K}$ matrix depends weakly on the center-of-mass momentum of the nucleon, and the Λ particle one uses the value of the $\mathcal{K}$ matrix for zero center-of-mass momentum.

By introducing the wave function for the relative motion of the nucleon and the Λ particle $\Psi_{\mathbf{k}}$ ($\mathbf{k}$ is the relative momentum), one obtains for V

$$V=[4/(2\pi)^3](M_N^*/\mu^*)^3\int_0^{\mu^* k_F/M_N^*} d\mathbf{k}\int d\mathbf{r}\exp(-i\mathbf{kr}) \times[v_{\Lambda C}(r)+v_{\Lambda A}(r)]\Psi_{\mathbf{k}}(\mathbf{r}), \quad (21)$$

where the reduced effective mass μ^* is given by

$$1/\mu^*=1/M_N^*+1/M_\Lambda^*, \quad (22)$$

and where M_N^*, M_Λ^* are the effective masses of the nucleon and the Λ particle, respectively.

The main approximation consists in replacing $\Psi_{\mathbf{k}}(\mathbf{r})$ in Eq. (21) by the wave function for a pure hard-core interaction. Its S-wave part is the known solution of the Bethe-Goldstone[17] equation:

$$[\Psi_{\mathbf{k}}(\mathbf{r})]_S\cong\Psi_{\mathrm{BG}}(k,r) =[A(k)/kr]\Big\{\sin k(r-c) +(1/\pi)\Big(\int_0^r dr'\sin k(r-r') \times\Big[\frac{\sin k_F(r'+c)}{(r'+c)}-\frac{\sin k_F(r'-c)}{(r'-c)}\Big]\Big)\Big\}, \quad (23)$$

where

$$1/A(k)=\cos kc+(1/\pi)\{\sin kc(\mathrm{Ci}[c(k_F+k)] -\mathrm{Ci}[c(k_F-k)])-\cos kc(\mathrm{Si}[c(k_F+k)] +\mathrm{Si}[c(k_F-k)])\}, \quad (24)$$

where Si and Ci are defined as in Jahnke-Emde.[18]

The S-wave part contribution to V can be easily calculated. In evaluating the hard-core part, one uses the relation

$$rv_{\Lambda C}(r)\Psi_{\mathrm{BG}}(k,r)\cong(\hbar^2/2\mu^*)A(k)\delta(r-c). \quad (25)$$

An approximate calculation of the higher l contributions to V is discussed in detail in I.

B. Expression for V_R

From the equations for $K(A)$ and $K(A+1_\Lambda)$, one obtains after some algebra,

$$(\mathbf{m}_1\mathbf{m}_2|K(A+1_\Lambda)-K(A)|\mathbf{m}_1\mathbf{m}_2) =\sum_{>k_F}\sum_{\mathbf{k}_1}\sum_{>k_F}{}_{\mathbf{k}_2}(\mathbf{m}_1\mathbf{m}_2|K(A)|\mathbf{k}_1\mathbf{k}_2) \times\delta\binom{1}{-}{e}(\mathbf{k}_1\mathbf{k}_2|K(A+1_\Lambda)|\mathbf{m}_1\mathbf{m}_2), \quad (26)$$

with

$$\delta(1/e)=[e_\Lambda(m_1)+e_\Lambda(m_2)-e_\Lambda(k_1)-e_\Lambda(k_2)]^{-1} -[e(m_1)+e(m_2)-e(k_1)-e(k_2)]^{-1} \cong-[e(m_1)+e(m_2)-e(k_1)-e(k_2)]^{-2} \times\{[e_\Lambda(m_1)-e(m_1)]+[e_\Lambda(m_2)-e(m_2)] -[e_\Lambda(k_1)-e(k_1)]-[e_\Lambda(k_2)-e(k_2)]\}, \quad (27)$$

where $e(k)$ and $e_\Lambda(k)$ are the single-particle model energies of a nucleon with momentum $\hbar k$ in nuclear matter and in the nuclear matter +Λ-particle system, respectively.

Now we introduce the following approximations discussed in detail by Brueckner *et al.*[14,15] First, we apply the first iteration to Eq. (26), i.e., replace $K(A+1_\Lambda)$ by $K(A)$ on the right-hand side of this equation. Second, we approximate the differences in the single-particle energies by

$$e_\Lambda(m_i)-e(m_i)\cong(\mathbf{m}_i\mathbf{m}_\Lambda|\mathcal{K}|\mathbf{m}_i\mathbf{m}_\Lambda), \quad (28)$$

$$e_\Lambda(k_i)-e(k_i)\cong(\mathbf{k}_i\mathbf{m}_\Lambda|\mathcal{K}_1|k_i\mathbf{m}_\Lambda), \quad (29)$$

where $i=1, 2$. The contribution of the Λ-nucleon interaction to the single-particle energy of a nucleon below the Fermi surface, Eq. (28), is determined by the on-energy-shell $\mathcal{K}$ matrix. However, for nucleon in excited states ($k_i>k_F$), one has to use the off-energy-shell reaction matrix denoted in Eq. (29) by $\mathcal{K}_1$. This well-known point of the Brueckner theory is discussed, e.g., in Refs. 14 and 19.

The two approximations inserted into Eq. (26) allow us to write V_R, Eq. (20), in the form

$$V_R\cong V_{Rh}+V_{Rp}, \quad (30)$$

where

$$V_{Rh}=-12\sum_{\mathbf{m}_1}\sum_{\mathbf{m}_2}\sum_{\mathbf{k}_1}\sum_{\mathbf{k}_2}\{(\mathbf{m}_1\mathbf{m}_2|K|\mathbf{k}_1\mathbf{k}_2) \times(\mathbf{k}_1\mathbf{k}_2|K|\mathbf{m}_1\mathbf{m}_2)/[e(m_1)+e(m_2) -e(k_1)-e(k_2)]^2\}(\mathbf{m}_1\mathbf{m}_\Lambda|\mathcal{K}|\mathbf{m}_1\mathbf{m}_\Lambda), \quad (31)$$

$$V_{Rp}=12\sum_{\mathbf{m}_1}\sum_{\mathbf{m}_2}\sum_{\mathbf{k}_1}\sum_{\mathbf{k}_2}\{(\mathbf{m}_1\mathbf{m}_2|K|\mathbf{k}_1\mathbf{k}_2) \times(\mathbf{k}_1\mathbf{k}_2|K|\mathbf{m}_1\mathbf{m}_2)/[e(m_1)+e(m_2) -e(k_1)-e(k_2)]^2\}(\mathbf{k}_1\mathbf{m}_\Lambda|\mathcal{K}_1|\mathbf{k}_1\mathbf{m}_\Lambda), \quad (32)$$

where $m_1<k_F$, $m_2<k_F$, $k_1>k_F$, $k_2>k_F$, and where K stands for $K(A)$ (we shall use this simplified notation throughout the following considerations).

These are the hole (V_{Rh}) and particle (V_{Rp}) rearrangement energies of third order. In the case of a pure

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[18] E. Jahnke and F. Emde, *Tables of Functions with Formulas and Curves* (Dover Publications, New York, 1945), 4th ed.

[19] H. A. Bethe, B. H. Brandow, and A. G. Petschek, Phys. Rev. **129**, 225 (1963).

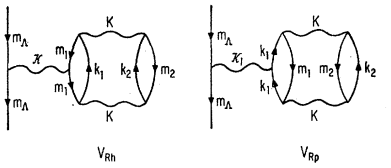

FIG. 1. Rearrangement energy diagrams.

nuclear matter, they have been considered by Brueckner *et al.*[14,15] who called them the effective mass rearrangement terms. In the case of a finite nucleus they were discussed in Ref. 16 (where V_{Rh} was denoted by V_{st} and V_{Rp} by V_p). The corresponding diagrams are shown in Fig. 1.

III. CALCULATION OF V_{Rh}

Let us first write the equation for the nucleon-nucleon reaction matrix K with the help of the wave operator Ω_N ($K=v_{NN}\Omega_N$) (Ref. 20):

$$(\mathbf{k}_1\mathbf{k}_2|\Omega_N|\mathbf{m}_1\mathbf{m}_2)=(\mathbf{k}_1\mathbf{k}_2|\mathbf{m}_1\mathbf{m}_2)+(\mathbf{k}_1\mathbf{k}_2|K|\mathbf{m}_1\mathbf{m}_2)/[e(m_1)+e(m_2)-e(k_1)-e(k_2)]. \quad (33)$$

This equation enables us to write Eq. (31) for V_{Rh} in the form

$$V_{Rh}=-12\sum_{\mathbf{m}_1}\sum_{\mathbf{m}_2}\sum_{\mathbf{k}_1}\sum_{\mathbf{k}_2}|(\mathbf{k}_1\mathbf{k}_2|\Omega_N-1|\mathbf{m}_1\mathbf{m}_2)|^2\times(\mathbf{m}_1\mathbf{m}_\Lambda|\mathcal{K}|\mathbf{m}_1\mathbf{m}_\Lambda). \quad (34)$$

In the relative and center-of-mass coordinates

$$\begin{aligned}\mathbf{K}&=\mathbf{k}_1+\mathbf{k}_2, \quad \mathbf{k}=(\mathbf{k}_1-\mathbf{k}_2)/2,\\ \mathbf{M}&=\mathbf{m}_1+\mathbf{m}_2, \quad \mathbf{m}=(\mathbf{m}_1-\mathbf{m}_2)/2,\end{aligned} \quad (35)$$

we have

$$(\mathbf{k}_1\mathbf{k}_2|\Omega_N-1|\mathbf{m}_1\mathbf{m}_2)=\delta_{\mathbf{KM}}(\mathbf{k}|\Omega_N-1|\mathbf{m})_\mathbf{M}, \quad (36)$$

where the matrix element on the right-hand side depends on $\mathbf{M}$. However, this dependence is weak and we shall use the approximation

$$(\mathbf{k}|\Omega_N-1|\mathbf{m})_\mathbf{M}\cong(\mathbf{k}|\Omega_N-1|\mathbf{m})_{\mathbf{M}=0}\equiv(\mathbf{k}|\Omega_N-1|\mathbf{m}). \quad (37)$$

Let us now introduce the wave function of the relative motion of the two nucleons (with $\mathbf{M}=0$):

$$(\mathbf{k}|\Psi_\mathbf{m}{}^N)=(\mathbf{k}|\Omega_N|\mathbf{m}), \quad (38)$$

and denote by $\chi_\mathbf{m}{}^N$ the difference between $\Psi_\mathbf{m}{}^N$ and the plane wave $\varphi_\mathbf{m}$.

$$(\mathbf{k}|\chi_\mathbf{m}{}^N)=(\mathbf{k}|\Omega_N-1|\mathbf{m}), \quad (39)$$

or, in configuration space,

$$\chi_\mathbf{m}{}^N(\mathbf{r})=\Psi_\mathbf{m}{}^N(\mathbf{r})-e^{i\mathbf{m}\mathbf{r}}. \quad (40)$$

Now, the summation over $\mathbf{k}_1$ and $\mathbf{k}_2$ in Eq. (34) can be transformed into an integration in configuration space[21]:

$$\sum_{\mathbf{k}_1}\sum_{\mathbf{k}_2}|(\mathbf{k}_1\mathbf{k}_2|\Omega_N-1|\mathbf{m}_1\mathbf{m}_2)|^2=\frac{1}{\Omega_V}\int d\mathbf{r}|\chi_\mathbf{m}{}^N(\mathbf{r})|^2. \quad (41)$$

The integral in Eq. (41) represents the correlation volume of the two interacting nucleons, and it is slowly varying with the relative momentum m. For example, if one approximates $\Psi_\mathbf{m}{}^N$ by the solution of the Bethe-Goldstone equation for the S wave [i.e., one neglects the attractive part of v_{NN} in Eq. (10) and the interaction in D state] one finds[22]

$$\int d\mathbf{r}|\chi_\mathbf{m}{}^N|^2=\left\{\begin{matrix}1.1\\1.2\\1.4\end{matrix}\right\}\mathrm{F}^3 \quad \text{for} \quad \frac{m}{k_F}=\left\{\begin{matrix}0\\0.5\\0.7\end{matrix}\right. \quad (42)$$

Hence, we shall make the following approximation:

$$\begin{aligned}\int d\mathbf{r}|\chi_\mathbf{m}{}^N|^2&\cong\int d\mathbf{r}|\chi^N|^2\\ &\equiv\int d\mathbf{m}_1\int d\mathbf{m}_2\int d\mathbf{r}|\chi_m{}^N|^2\Big/\int d\mathbf{m}_1\int d\mathbf{m}_2\\ &=\frac{24}{k_F{}^3}\int_0^{k_F}dmm^2\left\{1-\frac{3}{2}\frac{m}{k_F}+\frac{1}{2}\left(\frac{m}{k_F}\right)^3\right\}\\ &\quad\times\int d\mathbf{r}|\chi_\mathbf{m}{}^N|^2, \quad (43)\end{aligned}$$

i.e., we shall use an average value of the correlation volume over the Fermi sea.

With the approximation (43), we have, according to Eq. (34)

$$V_{Rh}=-12(1/\Omega_V)\int d\mathbf{r}|\chi^N|^2\times\sum_{\mathbf{m}_1}(\mathbf{m}_1\mathbf{m}_\Lambda|\mathcal{K}|\mathbf{m}_1\mathbf{m}_\Lambda)\sum_{\mathbf{m}_2}. \quad (44)$$

Since $\sum_{\mathbf{m}_2}=A/4$, we get with the help of Eq. (19)

$$V_{Rh}=-\tfrac{3}{4}\rho\left|\int d\mathbf{r}\chi^N(\mathbf{r})\right|^2V, \quad (45)$$

i.e., we get the result that the hole rearrangement energy is simply proportional to the single-particle model potential V, and the whole problem of calculating V_{Rh} is thus reduced to calculating the (averaged) correlation volume.

Actually, in calculating the correlation volume, it is

[20] The method of calculating V_{Rh} presented in this section has been applied first in Ref. 16.

[21] Notice our normalization: $(\mathbf{r}|\mathbf{m})=\Omega_V{}^{-1}\exp[i\mathbf{m}\mathbf{r}]$, where Ω_V is the volume of the periodicity box.

[22] Right at the Fermi surface ($m=k_F$), the solution of the Bethe-Goldstone equation "blows up" and the correlation volume becomes infinite. This, however, has no serious effect on V_{Rh} since the states around $m=k_F$ have a negligible weight.

convenient to use the relation

$$\int d\mathbf{r}|\chi_{\mathbf{m}}{}^{N}(\mathbf{r})|^2=(1/2\pi)^3\int d\mathbf{k}|\tilde{\chi}_{\mathbf{m}}{}^{N}(\mathbf{k})|^2, \quad (46)$$

where $\tilde{\chi}_{\mathbf{m}}{}^{N}(\mathbf{k})$ is the Fourier transform of $\chi_{\mathbf{m}}{}^{N}(\mathbf{r})$.

$$\frac{1}{\Omega_V}\tilde{\chi}_{m}{}^{N}(\mathbf{k})=\frac{1}{\Omega_V}\int d\mathbf{r}e^{-i\mathbf{k}\mathbf{r}}\chi_{m}{}^{N}(\mathbf{r})=(\mathbf{k}|\Omega_N-1|\mathbf{m}). \quad (47)$$

By writing Eq. (33) in the relative coordinates and applying the effective-mass approximation to the single-particle energies e, we find

$$\tilde{\chi}_{\mathbf{m}}{}^{N}(\mathbf{k})=\frac{M_N{}^*}{\hbar^2}\int d\mathbf{r}e^{-i\mathbf{k}\mathbf{r}}v_{NN}\Psi_{m}{}^{N}(\mathbf{r})/(m^2-k^2). \quad (48)$$

In applying Eq. (48) we shall approximate $\Psi_{\mathbf{m}}{}^{N}$ by the wave function for the pure hard-core interaction v_{NC}, acting in S state only. [The last approximation neglects the D-state interaction, since our v_{NN}, Eq. (9), acts in even l states only.] This means we put

$$\Psi_{\mathbf{m}}{}^{N}(\mathbf{r})\cong\Psi_{\mathrm{BG}}(m,r), \quad (49)$$

where Ψ_{BG} is the Bethe-Goldstone wave function introduced in Sec. IIA. Notice that Ψ_{BG} depends only on the hard-core radius and hence is the same in the nucleon-nucleon as in the Λ-nucleon case. The only difference between the two cases consists in replacing Eq. (25) by

$$rv_{NC}(r)\Psi_{\mathrm{BG}}(m,r)\cong(\hbar^2/M_N{}^*)A(m)\delta(r-c). \quad (50)$$

With the help of Eq. (50) we finally get

$$\tilde{\chi}_{\mathbf{m}}{}^{N}(k)\cong\frac{4\pi}{k}\left\{A(m)\sin kc+\frac{M_N{}^*}{\hbar^2}\times\int drr\sin krv_{NA}(r)\Psi_{\mathrm{BG}}(m,r)\right\}\Big/(m^2-k^2). \quad (51)$$

Let us notice that, similar to Eq. (43), we can introduce an average value of $|\tilde{\chi}_{\mathbf{m}}{}^{N}(k)|^2$ over the Fermi sea.

$$|\tilde{\chi}^{N}(k)|^2=\frac{24}{k_F{}^3}\int_0^{k_F}dmm^2\left\{1-\frac{3}{2}\left(\frac{m}{k_F}\right)+\frac{1}{2}\left(\frac{m}{k_F}\right)^3\right\}|\tilde{\chi}_{\mathbf{m}}{}^{N}(k)|^2, \quad (52)$$

and combine Eqs. (43) and (46) to

$$\int d\mathbf{r}|\chi^{N}(r)|^2=(1/2\pi)^3\int d\mathbf{k}|\tilde{\chi}^{N}(k)|^2. \quad (53)$$

IV. CALCULATION OF V_{Rp}

The main problem here is to determine the off-energy-shell Λ-nucleon reaction matrix $\mathcal{K}_1$. In the first part of this section, we shall calculate $\mathcal{K}_1$ in an approximate way applied in the pure nuclear case by Bethe and his collaborators.[19]

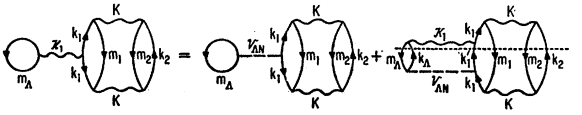

FIG. 2. Diagramatic representation of the equation for $\mathcal{K}_1$.

The matrix element $(\mathbf{k}_1\mathbf{m}_\Lambda|\mathcal{K}_1|\mathbf{k}_1\mathbf{m}_\Lambda)$ entered into Eq. (32) as the contribution of Λ-nucleon interaction to the single-particle energy of a nucleon in an excited virtual state of momentum $\mathbf{k}_1$. This matrix element contains the sum of all the corresponding ladder diagrams and the appropriate equation for $(\mathbf{k}_1\mathbf{m}_\Lambda|\mathcal{K}_1|\mathbf{k}_1\mathbf{m}_\Lambda)$ is shown diagrammatically in Fig. 2. This diagrammatic equation takes the following analytical form:

$$(\mathbf{k}_1\mathbf{m}_\Lambda|\mathcal{K}_1|\mathbf{k}_1\mathbf{m}_\Lambda)=v_\Lambda|\mathbf{k}_1\mathbf{m}_\Lambda)+\sum_{k_1'}\sum_{k_\Lambda}v_{\Lambda N}|\mathbf{k}_1'\mathbf{k}_\Lambda)\times\frac{1}{e(k_1)+\bar{e}(m_\Lambda)-e(k_1')-\bar{e}(k_\Lambda)-\Delta(k_1k_2;m_1m_2)}\times(\mathbf{k}_1'\mathbf{k}_\Lambda|\mathcal{K}_1|\mathbf{k}_1\mathbf{m}_\Lambda), \quad (54)$$

where

$$\Delta(k_1k_2;m_1m_2)=e(k_1)+e(k_2)-e(m_1)-e(m_2), \quad (55)$$

with e and $\bar{e}$ denoting the single-particle energies of nucleons and Λ particle, respectively. The energy denominator in Eq. (54) is equal to the negative excitation energy of the system in the intermediate state indicated in Fig. 2 by the dotted line.

The sum over $\mathbf{k}_1'$ in Eq. (54) is restricted by the exclusion principle to $k_1'>k_F$. However, the range of k_1 is $k_1>k_F$ and within this range the exclusion principle is less effective compared to the off-energy-shell effect represented by Δ. Hence, we shall ignore the exclusion principle in Eq. (35), i.e., we shall consider the $\mathbf{k}_1'$ sum to be extended over the whole momentum space.

With the help of the wave-function operator Ω_1 defined by

$$\mathcal{K}_1=v_{N\Lambda}\Omega_1, \quad (56)$$

we can rewrite Eq. (54) in the form

$$(\mathbf{k}_1'\mathbf{k}_\Lambda|\Omega_1-1|\mathbf{k}_1\mathbf{m}_\Lambda)=(\mathbf{k}_1'\mathbf{k}_\Lambda|v_{N\Lambda}\Omega_1|\mathbf{k}_1\mathbf{m}_\Lambda)/[e(k_1)+\bar{e}(m_\Lambda)-e(k_1)-\bar{e}(k_\Lambda)-\Delta(k_1k_2,m_1m_2)]. \quad (57)$$

For the single-particle energies we shall apply the effective-mass approximation

$$\bar{e}(k)\cong(\hbar^2/2M_\Lambda{}^*)k^2+\text{const},\qquad e(k)\cong(\hbar^2/2M_N{}^*)k^2+\text{const}. \quad (58)$$

With the Pauli principle being ignored and with the approximation (58), the center-of-mass motion can be

separated. Let us introduce the center-of-mass and relative momenta

$$\begin{aligned}\mathbf{K}&=\mathbf{k}_1+\mathbf{m}_\Lambda,\\ \mathbf{K}'&=\mathbf{k}_1'+\mathbf{k}_\Lambda,\\ \mathbf{k}&=(\mu^*/M_N^*)\mathbf{k}_1-(\mu^*/M_\Lambda^*)\mathbf{m}_\Lambda,\\ \mathbf{k}'&=(\mu^*/M_N^*)\mathbf{k}_1'-(\mu^*/M_\Lambda^*)\mathbf{k}_\Lambda.\end{aligned}\tag{59}$$

Actually, we have $\mathbf{m}_\Lambda=0$ and

$$\mathbf{k}=\mu^*\mathbf{k}_1/M_N^*.\tag{60}$$

For the energy denominator in Eq. (57) we get

$$e(k_1)+\bar{e}(m_\Lambda)-e(k_1')-\bar{e}(k_\Lambda)=(\hbar^2/2\mu^*)(k^2-k'^2),\tag{61}$$

$$\begin{aligned}&\Delta(k_1k_2,m_1m_2)\\ &\quad=(\hbar^2/M_N^*)[k_1^2+\mathbf{m}_1\mathbf{m}_2-\mathbf{m}_1\mathbf{k}_1-\mathbf{m}_2\mathbf{k}_1].\end{aligned}\tag{62}$$

In the last equation we have used the momentum conservation

$$\mathbf{k}_2=\mathbf{m}_1+\mathbf{m}_2-\mathbf{k}_1.\tag{63}$$

We shall approximate Δ by its angle average.

$$\Delta(k_1k_2,m_1m_2)\cong(\hbar k_1)^2/M_N^*.\tag{64}$$

Let us now introduce the wave function

$$(\mathbf{k}_1'\mathbf{k}_\Lambda|\Omega_1|\mathbf{k}_1\mathbf{m}_\Lambda)=\delta_{\mathbf{K}',\mathbf{K}}(k'|\Psi_{1,k})\tag{65}$$

and

$$\chi_{1,k}=\Psi_{1,k}-\varphi_k,\tag{66}$$

where φ_k is a plane wave. If we transform Eq. (57) [or (54)] into configuration space and take into account Eqs. (60), (61), (64), we finally get

$$(\hbar^2/2\mu^*)(\nabla^2-\gamma^2)\chi_{1,k}(\mathbf{r})=v_{\Lambda N}\Psi_{1,k}(\mathbf{r}),\tag{67}$$

where $\mathbf{r}$ is the relative Λ-nucleon coordinate and

$$\gamma^2=(2(M_N^*/M_\Lambda^*)+1)k^2.\tag{68}$$

The matrix element of $\mathcal{K}_1$ is according to Eq. (56) given by

$$(\mathbf{k}_1\mathbf{m}_\Lambda|\mathcal{K}_1|\mathbf{k}_1\mathbf{m}_\Lambda)=\mathcal{K}_1(k)/\Omega_V\tag{69}$$

with

$$\mathcal{K}_1(k)=\int d\mathbf{r}e^{-i\mathbf{k}\mathbf{r}}v_{\Lambda N}(r)\Psi_{1,k}(\mathbf{r}).\tag{70}$$

Hence, to calculate the off-energy-shell reaction matrix $\mathcal{K}_1$ which enters into Eq. (32) for V_{Rp}, we have to solve Eq. (67) for $\Psi_{1,k}$ and then calculate the integral in Eq. (70).

Instead of solving Eq. (67) exactly we shall apply the modified Born approximation discussed by Bethe *et al.*[19] First we decompose $\Psi_{1,k}$ into partial waves

$$\Psi_{1,k}(\mathbf{r})=\sum_L(2L+1)i^LR_L(r)P_L(\hat{k}\hat{r}).\tag{71}$$

This gives us a corresponding decomposition of $\mathcal{K}_1$,

$$\mathcal{K}_1(k)=4\pi\sum_L(2L+1)\int_0^\infty drr^2j_L(kr)v_{\Lambda N}R_L.\tag{72}$$

With the help of Eq. (67) one can easily derive (see Ref. 19 for a detailed derivation) the relation

$$\begin{aligned}&\int_0^\infty drr^2j_L(kr)v_{\Lambda N}R_L\\ &=\frac{\hbar^2}{2\mu^*}\Big\{(k^2+\gamma^2)\int_0^c drr^2j_L(kr)^2\\ &\qquad+c^2j_L(kc)\Big(\frac{d}{dr}R_L^0\Big)_c\Big\}+\int_c^\infty drr^2R_L^0v_{\Lambda N}R_L,\end{aligned}\tag{73}$$

where the function R_L^0,

$$R_L^0(r)=j_L(kr)-\frac{j_L(kc)}{h_L^{(1)}(i\gamma c)}h_L^{(1)}(i\gamma r),\tag{74}$$

is the R_L function for a pure hard-core interaction.

As is shown in Ref. 19, after inserting (73) into Eq. (70), one can perform most of the summations over L exactly with the result

$$\mathcal{K}_1(k)=\mathcal{K}_1(k)_{\text{core}}+\mathcal{K}_1(k)_{\text{attractive}},\tag{75}$$

with

$$\begin{aligned}\mathcal{K}_1(k)_{\text{core}}\cong&4\pi(\hbar^2/2\mu^*)\{\tfrac{1}{3}(k^2+\gamma^2)c^3\\ &+c[1+\gamma c+\tfrac{1}{3}(kc)^2/(1+\gamma c)]\},\end{aligned}\tag{76}$$

$$\mathcal{K}_1(k)_{\text{attractive}}=4\pi\sum_L(2L+1)\int_c^\infty drr^2R_L^0v_{\Lambda N}R_L.\tag{77}$$

The modified Born approximation which we want to apply consists in replacing R_L in Eq. (75) by R_L^0. That means we shall use the approximation

$$\mathcal{K}_1(k)_{\text{attractive}}\cong4\pi\sum_L(2L+1)\int_c^\infty drr^2R_L^0v_{\Lambda N}R_L^0.\tag{78}$$

Equations (75), (76), (78) determine our approximation to $\mathcal{K}_1$. With the help of Eq. (60) we finally get $\mathcal{K}_1$ as a function of k_1.

$$\mathcal{K}_1(k)=\mathcal{K}_1(\mu^*k_1/M_N^*).\tag{79}$$

To calculate V_{Rp} given by Eq. (32), we proceed in the same way as in Sec. III. With the help of the approximation (37), we thus get

$$\begin{aligned}V_{Rp}=&(12/\Omega_V^3)\sum_{\mathbf{m}_1}\sum_{\mathbf{m}_2}\sum_{\mathbf{k}_1}\sum_{\mathbf{k}_2}\delta_{\mathbf{K},\mathbf{M}}|\tilde{\chi}_{\mathbf{m}}^N(\mathbf{k})|^2\\ &\times\mathcal{K}_1(\mu^*k_1/M_N^*)\\ =&(12/\Omega_V^3)\sum_{\mathbf{m}_1}\sum_{\mathbf{m}_2}\sum_{\mathbf{k}_1}\Big|\tilde{\chi}_{\mathbf{m}}^N\Big(\mathbf{k}_1-\frac{\mathbf{m}_1+\mathbf{m}_2}{2}\Big)\Big|^2\\ &\times\mathcal{K}_1(\mu^*k_1/M_N^*).\end{aligned}\tag{80}$$

Because of the Fourier transform $\tilde{\chi}$ of the function χ, most of the contribution to V_{Rp} comes from $k_1 \gtrsim 1.5k_F$ where we can neglect $(\mathbf{m}_1+\mathbf{m}_2)/2$ compared to $\mathbf{k}_1$. Hence we put in Eq. (80)

$$\tilde{\chi}_{\mathrm{m}}{}^{N}\left(\mathbf{k}_1-\frac{\mathbf{m}_1+\mathbf{m}_2}{2}\right)\cong\tilde{\chi}_{\mathrm{m}}{}^{N}(\mathbf{k}_1),$$

and get

$$V_{Rp}=(12/\Omega_V{}^3)\sum_{\mathbf{k}_1}\{\sum_{\mathrm{m}_1}\sum_{\mathrm{m}_2}|\tilde{\chi}_{\mathrm{m}}{}^{N}(\mathbf{k}_1)|^2\}\times\mathcal{K}_1(\mu^*k_1/M_N^*). \quad (81)$$

As in Sec. III, we shall use the approximate equation (51) for $\tilde{\chi}_{\mathrm{m}}{}^{N}(\mathbf{k}_1)=\tilde{\chi}_{\mathrm{m}}{}^{N}(k_1)$. Now the summation over $\mathbf{m}_1$ and $\mathbf{m}_2$ produces the average $|\tilde{\chi}^N(k_1)|^2$ defined in Eq. (52).

$$(12/\Omega_V{}^2)\sum_{\mathrm{m}_1}\sum_{\mathrm{m}_2}|\tilde{\chi}_{\mathrm{m}}{}^{N}(k_1)|^2=\tfrac{3}{4}\rho^2|\tilde{\chi}^N(k_1)|^2, \quad (82)$$

and hence for V_{Rp} we get finally

$$V_{Rp}=\tfrac{3}{4}\rho^2\left(\frac{1}{2\pi}\right)^3 4\pi\int dk_1k_1{}^2|\tilde{\chi}^N(k_1)|^2\mathcal{K}_1(\mu^*k_1/M_N^*). \quad 83)$$

V. ESTIMATE OF M_Λ^*

We shall present here a simplified estimate of the effective mass of the Λ particle for high-energy excitations where the hard core in the Λ-nucleon interaction plays the predominant role. Our estimate differs from that of Bethe *et al.*[19] only as far as we have in our case two different particles, hence, a different reduced mass appears, and there is no exchange contribution (which anyhow has been neglected in Ref. 19). One also has to distinguish in our case between the nucleon-nucleon and Λ-nucleon reaction matrices.

The Λ-nucleon contribution to the energy $E(A+1_\Lambda)$ of the nuclear matter $+\Lambda$-particle system is in the reaction-matrix theory given by the diagram (a) in Fig. 3. By defining the single-particle energy of a Λ particle which enters into the reaction-matrix equation by the equation

$$\bar{e}(k_\Lambda)=\frac{\hbar^2k_\Lambda{}^2}{2M_\Lambda}+4\sum_{<k_F}{}_{\mathrm{m}_N'}(\mathbf{m}_N'\mathbf{k}_\Lambda|\mathcal{K}_1|\mathbf{m}_N'\mathbf{k}_\Lambda), \quad (84)$$

one automatically includes into the diagram (a) in Fig. 3 the Λ-particle self-energy diagrams of the type shown in Fig. 3(b). The off-energy-shell reaction matrix $\mathcal{K}_1$ has been discussed in Sec. IV. The only difference is that now we are interested in the diagonal matrix element in the state in which the nucleon wave vector $m_N'<k_F$ and the Λ-particle wave vector $k_\Lambda>0$. In the following discussion we shall restrict ourselves to big values of k_Λ and accordingly we shall neglect the exclusion principle in the equation for $\mathcal{K}_1$ as was also done in Sec. IV.

One can now repeat the whole procedure of determining $\mathcal{K}_1$ of Sec. IV taking, however, into account the different values of the nucleon and Λ-particle momenta. Instead of Eq. (55) the off-energy-shell shift Δ is now given by

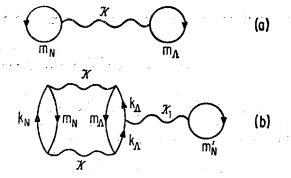

FIG. 3. Diagramatic representation of the Λ-nucleon contribution to $E(A+1_\Lambda)$: (a) first-order diagram, (b) Λ self-energy diagram of third order.

$$\Delta(k_\Lambda k_N;m_\Lambda m_N)=\bar{e}(k_\Lambda)+e(k_N)-\bar{e}(m_\Lambda)-e(m_N). \quad (85)$$

By applying the momentum conservation

$$\mathbf{k}_N=\mathbf{m}_\Lambda+\mathbf{m}_N-\mathbf{k}_\Lambda=\mathbf{m}_N-\mathbf{k}_\Lambda, \quad (86)$$

($\mathbf{m}_\Lambda=0$) and the effective-mass approximation, Eq. (58), we get

$$\Delta=(\hbar^2/2\mu^*)k_\Lambda{}^2-2(\hbar^2/2M_N^*)\mathbf{m}_N\mathbf{k}_\Lambda\cong(\hbar^2/2\mu^*)k_\Lambda{}^2, \quad (87)$$

where in the last step we have replaced Δ by its angle average.

Let us introduce the relative Λ-nucleon momentum

$$\mathbf{p}=\mu^*[\mathbf{m}_N'/M_N^*-\mathbf{k}_\Lambda/M_\Lambda^*]. \quad (88)$$

For big values of k_Λ ($k_\Lambda\gg k_F$) we have

$$k_\Lambda{}^2\cong(M_\Lambda^*/\mu^*)^2p^2, \quad (89)$$

and we can write Eq. (87) in the form

$$\Delta\cong\frac{\hbar^2}{2\mu^*}(M_N^*/\mu^*)p^2. \quad (90)$$

Instead of Eqs. (69), (70) of Sec. IV we introduce

$$(\mathbf{m}_N'\mathbf{k}_\Lambda|\mathcal{K}_1|\mathbf{m}_N'\mathbf{k}_\Lambda)=(1/\Omega_V)\int d\mathbf{r}e^{-i\mathbf{p}\mathbf{r}}v_{\Lambda N}(r)\Psi_{1,\mathrm{p}}(\mathbf{r}), \quad (91)$$

and instead of Eq. (67) we now have

$$(\hbar^2/2\mu^*)(\nabla^2-\gamma_p{}^2)(\Psi_{1,\mathrm{p}}-e^{i\mathbf{p}\mathbf{r}})=v_{\Lambda N}\Psi_{1,\mathrm{p}}, \quad (92)$$

where

$$\gamma_p{}^2=(2+M_\Lambda^*/M_N^*)(M_\Lambda^*/M_N^*)p^2. \quad (93)$$

For big values of p (or k_Λ) we can approximate $v_{\Lambda N}$ in Eq. (92) by its repulsive core part $v_{\Lambda C}$. Considering that

$$v_{\Lambda C}\Psi_{1,\mathrm{p}}=0 \text{ for } r>c, \quad \Psi_{1,\mathrm{p}}=0 \text{ for } r<c, \quad (94)$$

we get from Eq. (92)

$$v_{\Lambda C}\Psi_{1,\mathrm{p}}=(\hbar^2/2\mu^*)(p^2+\gamma_p{}^2)e^{i\mathbf{p}\mathbf{r}}. \quad (95)$$

By inserting (95) into Eq. (91), we get

$$(\mathbf{m}_N'\mathbf{k}_\Lambda|\mathcal{K}_1|\mathbf{m}_N'\mathbf{k}_\Lambda)\cong(1/\Omega_V)(\hbar^2/2\mu^*)(p^2+\gamma_p{}^2)(4\pi/3)c^3. \quad (96)$$

Hence, according to Eq. (58) we have for big values of k_Λ,

$$\bar{e}(k_\Lambda)\cong(\hbar^2k_\Lambda{}^2/2M_N)+\rho(\hbar^2/2\mu^*)(p^2+\gamma_p{}^2)(4\pi/3)c^3. \quad (97)$$

A comparison with Eq. (58) gives finally

$$(c/r_0)^3(1+M_\Lambda^*/M_N^*)+M_\Lambda^*/M_\Lambda=1. \quad (98)$$

In obtaining Eq. (98) we have used Eq. (89), the definition of γ_p, Eq. (93), and the relation between r_0 and ρ given in Eq. (4).

With the numerical values of r_0, c, M_N^* introduced in Sec. I, Eq. (98) gives

$$M_\Lambda^*/M_\Lambda=0.887. \quad (99)$$

VI. NUMERICAL RESULTS AND DISCUSSION

All the numerical computations have been performed on the CDC-1604 computer of the Computer Center of the University of California at San Diego.

Numerical values of all the quantities (M_N^*,ρ,v_{NN},$v_{\Lambda N}$) used in the computations are those described in Sec. I.

In our numerical calculations we have considered two cases: $M_\Lambda^*=M_\Lambda$ and $M_\Lambda^*=0.887M_\Lambda$.

The values of V in Table I in the case $M_\Lambda^*=M_\Lambda$ have been taken from II. Within the approximations of Sec. IIA, only the repulsive core [$v_{\Lambda C}$ part of Eq. (21)] contribution to V depends on the value of M_Λ^*.[23] Hence, to get V in the case $M_\Lambda^*=0.887M_\Lambda$, we only have to calculate the $v_{\Lambda C}$ contribution to V. This has been done for the S state according to Eqs. (21), (23), (25). The higher l contributions calculated with the help of the approximate expressions given in I are practically the same for both the values of M_Λ^*, and are very small compared to the S-state contribution. Hence, the whole difference between V in the case $M_\Lambda^*=M_\Lambda$ and $M_\Lambda^*=0.887M_\Lambda$ consists in an increased S-state contribution of $v_{\Lambda C}$ in the latter case.

The calculation of V_{Rh} has been reduced in Sec. III to the calculation of the correlation volume. This has been done by first calculating $\tilde{\chi}_{\mathbf{m}}^N(k)$ according to Eq. (51). Then the average $|\tilde{\chi}^N(k)|^2$ has been calculated according to Eq. (52). And finally Eq. (53) has been used to calculate the correlation volume with the result

$$\int d\mathbf{r}|\chi^N(r)|^2=1.16, \quad (100)$$

which, multiplied by the factor $\frac{3}{4}\rho$, Eq. (45), gives 0.15. This means that V_{Rh} is 15% of $-V$. A comparison of Eqs. (100) and (42) shows that the correction to the Bethe-Goldstone wave function due to the attractive part of v_{NN}, v_{NA} in Eq. (51), has little effect on the magnitude of the correlation volume.

The particle rearrangement term V_{Rp} has been calculated according to Eq. (83). The average $|\chi^N(k_1)|^2$ has been computed as in the case of V_{Rh}. The $\mathcal{K}_1$ matrix has been computed according to Eqs. (76), (77). The contribution of the attractive part of $\mathcal{K}_1$ depends on M_Λ^* only very weakly through the relation (79). We have included partial waves with $L\leq 4$ in calculating $\mathcal{K}_1(k)_{\text{attractive}}$.

[23] The $v_{\Lambda\Lambda}$ contribution to V does not depend on M_Λ^* at all if one uses the approximate form of $\Psi_{\mathbf{k}}$ given in Eq. (9) of II.

TABLE I. Numerical results in MeV. The $v_{\Lambda N}$ potential denoted by (a)–(c) corresponds to the values of b, V_0 given in Eqs. (8a)–(8c), respectively.

M_Λ/M_Λ^*	$v_{\Lambda N}$	V	V_{Rh}	V_{Rp}	V_R	$U=-B_\Lambda(\infty)$
1	(a)	−41.3	6.2	0.7	6.9	34.4
	(b)	−40.3	6.0	0.6	6.6	33.7
	(c)	−40.1	6.0	0.7	6.7	33.4
0.887	(a)	−38.2	5.7	1.2	6.9	31.3
	(b)	−37.2	5.5	1.0	6.5	30.7
	(c)	−37.0	5.5	1.1	6.6	30.4

As an example, we give the different contributions to V_{Rp} in the case of the $v_{\Lambda N}$ potential of Eq. (86) for $M_\Lambda^*=M_\Lambda$. The contributions of $\mathcal{K}_1(k)_{\text{attractive}}$ to V_{Rp} for different values of L, Eq. (77), are in this case

$$[V_{Rp}]_{\text{attractive}}=\begin{Bmatrix}-2.6\\-5.3\\-2.9\\-1.1\\-0.4\end{Bmatrix}\text{MeV} \quad \text{for} \quad L=\begin{Bmatrix}0\\1\\2.\\3\\4\end{Bmatrix} \quad (101)$$

The contribution of $\mathcal{K}_1(k)_{\text{core}}$ to V_{Rp} is in the same case

$$[V_{Rp}]_{\text{core}}=12.9\ \text{MeV}, \quad (102)$$

and the total V_{Rp} is

$$V_{Rp}=0.6\ \text{MeV}. \quad (103)$$

The situation in other cases is very much the same. To a high degree the repulsive and attractive contributions cancel each other. The net effect is a very small positive value of V_{Rp}. This is the result of the hard core being much more effective in the case of V_{Rp} than in the case of V_{Rh}. Namely, in the case of V_{Rp} the Λ-nucleon interaction occurs at high energy (nucleon in an excited state) and off the energy shell, whereas in the case of V_{Rh} the Λ-nucleon interaction occurs at low energy (nucleon in a state below the Fermi level) and on the energy shell.

The results of our numerical computations are shown in Table I. The most important correction to V is the rearrangement energy V_R, strictly speaking its hole rearrangement part V_{Rh}. The rearrangement energy, V_R, is not sensitive to the value of M_Λ^*. However, V is reduced appreciably if M_Λ is replaced by $M_\Lambda^*=0.887M_\Lambda$.

The results show that by including V_R into the calculation of $B_\Lambda(\infty)$ and by using a reasonably estimated value of $M_\Lambda^*<M_\Lambda$ one gets a very good agreement with the experimentally estimated value of $B_\Lambda(\infty)$. This agreement has been obtained with the Λ-nucleon potential which, used in II, gave the best fit to the Λ-nucleon scattering data.

In view of the approximations applied in the present paper the accuracy of our results certainly does not

exceed a few MeV. Also the calculation of V in I and II is approximate. In particular, the main contribution to V from the attractive part of $v_{\Lambda N}$, namely that in the P state, has been calculated in an approximation of uncertain accuracy.

Still our results show—at least qualitatively—that there is no serious discrepancy between the calculated and measured binding energy of a Λ-particle in heavy hypernuclei.

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Electron Bremsstrahlung in Scattering by Nuclear Magnetic Moments*†

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The cross section for electron bremsstrahlung in the presence of a magnetic dipole potential is considered, with dependence on photon polarization explicit. Modifications of a result due to Sarkar, to include nuclear spin effects, are derived, and the angular and energy distributions of the radiated quanta are obtained. The related process of pair production is discussed. The infrared divergence is eliminated in the same way as for the Coulomb potential.

I. INTRODUCTION

IN the scattering of electrons by a nucleus, the emission of photons depends on the nuclear magnetic moment as well as on the nuclear charge. Sarkar[1] has obtained the bremsstrahlung cross section corresponding to a spin-independent (i.e., classical) nuclear magnetic moment. It is the purpose of this paper to determine the effects of nuclear spin on the cross section, to obtain the angular and energy distributions of the radiated particles, and to show that, as in the Coulomb case, the infrared divergence is spurious.

The results presented parallel those of Bethe and Heitler,[2] and of Gluckstern, Hull, and Breit[3] for bremsstrahlung in the Coulomb field.

An electromagnetic potential is introduced to represent the nucleus

$$A_\nu(\mathbf{r})=(-\boldsymbol{\mu}\times\boldsymbol{\nabla},ieZ)r^{-1}, \tag{1.1}$$

where μ and Z are the nuclear magnetic moment and atomic number. The relative magnitude of the magnetic and Coulomb interactions with the electron is considered by Newton,[4] the ratio being

$$\mu|\mathbf{q}|/eZ=(|\mathbf{q}|\mu/\mu_N)/2McZ, \tag{1.2}$$

where M is the mass of the nucleon, μ_N the nuclear magneton, and $\mathbf{q}$ a momentum transfer characteristic of the scattering process. Evidently, magnetic scattering is of greatest importance for high-energy electrons, the effect decreasing with Z. Unless the momentum transfer is comparable with the nuclear mass, the existence of magnetic properties of the nucleus is almost completely masked by the nuclear charge.

The assumption that the nucleus does not recoil is admittedly unrealistic for very light nuclei, since it is necessary that the experiments be performed at high energies. The most serious violation of this approximation, scattering from the proton, has been considered by Berg and Lindner.[5]

It is interesting to note that polarized targets, suitable for scattering experiments, are currently under investigation.[6]

II. THE DIFFERENTIAL CROSS SECTION

The electromagnetic potential is treated in the first Born approximation. If $(\mathbf{p}_0,iE_0)$ denotes the four-momentum of the incident electron, $(\mathbf{p},iE)$ that of the electron after scattering, then the cross section for emission of a photon with momentum $\mathbf{k}$ and polarization direction $\hat{e}$, is[1]

$$d\sigma=(Z^2e^6/8\pi^2)(kdk/q^4)(p/p_0)\,\mathrm{Tr}(A^++B^+)\times(H+E)(A+B)(H_0+E_0)d\Omega d\Omega_k, \tag{2.1}$$

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